BEECHER CARLSON

Loss Dependence – A Portfolio Level View of Retained Exposures

White paper 2 of 3

June 2018

This is the second white paper in this series. If you missed the first paper, you can download a copy here. To demonstrate the importance of dependence between risks, we once again utilize a highly simplified corporate case study.

White paper 1: Beyond Expected Value – Considering Volatility
White paper 2: Loss Dependence – A Portfolio Level View of Retained Exposure
White paper 3: Corporate Risk Appetite and Program Structuring

COMPANY ABC ILLUSTRATIVE CASE STUDY

In our first paper, we reviewed how expected value is inadequate as a standalone measure of risk, and we made the case that loss volatility should also impact risk financing decisions. In the following example, we examine how correlations among risks may impact the overall volatility of an entity's retained loss exposure.

Large risk bearing organizations are typically concerned with more than one potential cause of loss or line of insurance coverage; however, we frequently see individual exposures considered in isolation with minimal regard for how correlation may impact the overall corporate risk profile. The simplified case below illustrates the impact that the consideration of dependence may have on a risk financing decision making process.

Company ABC is assessing the risk associated with two exposures: Risk X and Risk Y. Risk X is larger than Risk Y in expected value terms, but Risk Y possesses a proportionally larger amount of volatility.

*Please see the Beecher Carlson white paper on volatility for further background on confidence levels and volatility.

Confidence Level	Risk X	Risk Y
Expected	$66.5M	$40.5M
VaR(90%)	$79.6M	$54.0M
VaR(95%)	$84.0M	$59.2M
VaR(99%)	$93.1M	$70.2M

As shown, for Risk X the 99% Value-at-Risk ($93.1m) is approximately 40% above the expected value, while for Risk Y the 99% Value-at-Risk ($70.2m) is approximately 73% greater than expected.

SIMPLIFYING ASSUMPTIONS

1. As in the prior paper, we assume that a credible stochastic model has been estimated for each of our sources of risk.
2. We assume that both of our sources of risks are continuous. In other words, we assume that each risk can realize a loss value of any dollar amount greater than or equal to zero.
3. We also make the simplifying technical assumption that the probabilistic dependence relationship between the two sources of loss exposure can be described by a single rank correlation parameter. In technical terms, we assume that the bivariate joint loss distribution is meta-Gaussian. This simplification has been chosen to make the illustration easier to follow. In real world situations, the dependence relationships that exist between the different sources of risk that a corporation is exposed to may follow more complex patterns.
4. We assume that Company ABC retains both sources of risk in its captive and sets targeted capital based on the 99% Value-at-Risk measure. Stated differently, we assume that Company ABC is willing to tolerate a 1% chance that the assets in its captive will be insufficient to fund the realized value of its retained losses.

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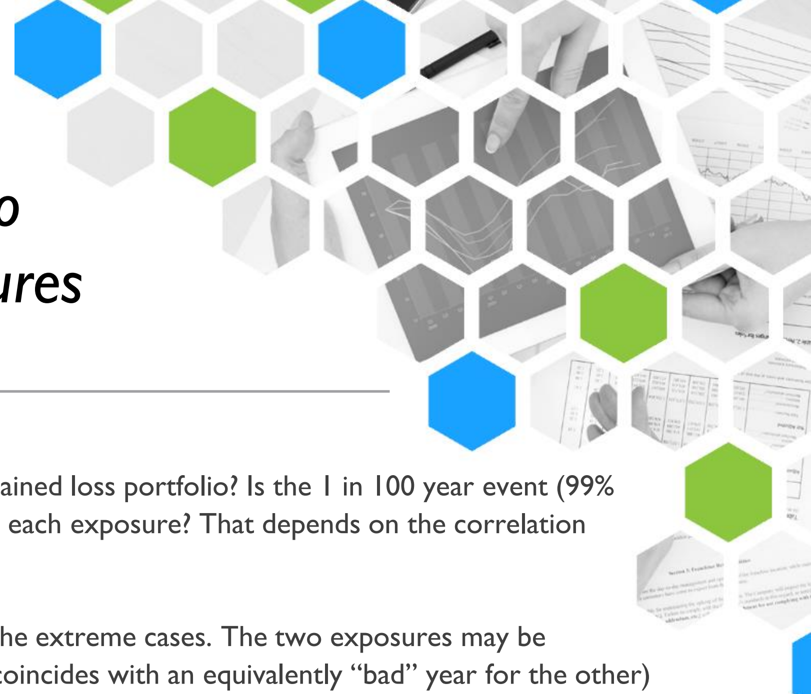

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What if we consider these two risks as part of an overall retained loss portfolio? Is the 1 in 100 year event (99% Value-at-Risk) simply the sum of the 1 in 100 year events for each exposure? That depends on the correlation between the two risks.

To see this numerically, it is helpful to begin by considering the extreme cases. The two exposures may be perfectly positively correlated (a "bad" year for one always coincides with an equivalently "bad" year for the other) – this would be considered a correlation of 1. Alternatively, the exposures may be completely independent. A bad year being realized for one exposure tells us absolutely nothing about our chances of observing a bad year for the other – a correlation of 0. These extremes yield the following results:

Conf Level	Risk X	Risk Y	Risk X + Risk Y	Risk(X+Y) Corr=1	Risk(X+Y) Corr=0
Expected	$66.5M	$40.5M	$107.0M	$107.0M	$107.0M
VaR(90%)	$79.6M	$54.0M	$133.6M	$133.6M	$126.1M
VaR(95%)	$84.0M	$59.2M	$143.2M	$143.2M	$132.7M
VaR(99%)	$93.1M	$70.2M	$163.3M	$163.3M	$145.3M

We can make several observations from the table above. First, the expected value of the combined risk is simply the sum of the expected values for the individual risks. Changing the correlation has no impact on the expected value of risks added together; however, when we are focusing on the Value-at-Risk measure this relationship no longer necessarily holds. If the risks are perfectly correlated (Corr=1), then the loss for the combined risk is equal to the sum of individual risks. This makes sense as adverse outcomes are tied together for the two risks. So, if a 1 in 100 year event happens for one, it happens for both.

But for uncorrelated coverages (Corr=0), we see a reduction in Value-at-Risk at higher confidence levels when the risks are combined. That is, an adverse outcome for one won't necessarily couple with an adverse outcome for the other. Thus, by combining these uncorrelated risks, the volatility of the overall program has been reduced.

In more 'realistic' scenarios, we typically see correlations that lie between the two extremes. Suppose that based on a review of historical development patterns, we estimate a correlation between Risk X and Risk Y of 25%. This means that there is some tendency for the percentile outcomes of X and Y to move in tandem – but that this tendency is far from perfect.

WHAT IS VALUE-AT-RISK?

For a given risk X, the Value-at-Risk measure at confidence level c% is denoted VaR(X, c%). VaR(X, c%) is the dollar amount such that there is a c% chance that losses incurred from this risk will be less than or equal to VaR(X, c%).

In insurance contexts, when our risk X is associated with an annual time horizon, VaR(X, c%) estimates are sometimes referred to as return period losses. In that case confidence level c% corresponds with a 1/(1-c%) year return period. For example, if c%=99% then VaR(X, c%) is the same as the 1/(1-99%)=100 year return period loss.

While this will lead to some reduction in volatility for the combined portfolio, the decrease will not be as significant as that seen for the zero correlation scenario, as is shown in the following table.

Conf Level	Risk X	Risk Y	Risk X + Risk Y	Risk(X+Y) Corr=.25
Expected	$66.5M	$40.5M	$107.0M	$107.0M
VaR(90%)	$79.6M	$54.0M	$133.6M	$128.3M
VaR(95%)	$84.0M	$59.2M	$143.2M	$136.1M
VaR(99%)	$93.1M	$70.2M	$163.3M	$150.1M

In the event that Company ABC computed a naïve estimate of the 99% Value-at-Risk for its retained portfolio by summing the Value-at-Risk estimates for X and Y, its computed captive capital requirement would be as follows:

Naïve Capital Requirement = VaR(X, 99%) + VaR(Y, 99%) – (E(X + Y))
= VaR(X, 99%) + VaR(Y, 99%) – (loss pick X + Y)
= $163.3M - $107.0M
= $56.3M

We have established above, however, that the true portfolio Value-at-Risk for the retained loss portfolio is less than the sum of the Value-at-Risk estimates for X and Y, due to the imperfect correlation between the two exposures. In properly incorporating this fact, Company ABC can maintain its target captive solvency probability of 99%, and re-compute its required captive capital amount as follows:

True Capital Requirement = VaR(X + Y, 99%) – (E(X + Y))
= VaR(X + Y, 99%) – (loss pick X + Y)
= $150.1M - $107.0M
= $43.1M

The difference between these two capital estimates can be viewed as a diversification benefit achieved by combining the two risks in a single portfolio and recognizing the lack of perfect correlation between them. Numerically, this can be computed as follows:

Diversification Benefit = Naïve Capital Requirement - True Capital Requirement = $13.2M

For Company ABC this $13.2M represents a material amount of funding that can be invested in its operations to help generate shareholder value, without raising the captive's chances of insolvency to an intolerable level.

This case study illustrated the mechanics of loss dependence in the highly simplified context of a world in which both relevant risks are unquestioningly retained by the company. Risk management decision making in the real world is much more complicated than this, and often involves complex trade-offs between risk and reward in the presence of many interdependent sources of risk. In the final paper from this series, we will delve into that topic when we discuss corporate risk appetite and its relationship to the program structuring process.

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